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Construction and analysis of non-Gaussian spatial models of neural spiking activity

Riccardo Barbieri^{a,*}, Loren M. Frank^a, Michael C. Quirk^b,
Victor Solo^c, Matthew A. Wilson^b, Emery N. Brown^a

^a*Neuroscience Statistics Research Laboratory, Department of Anesthesia and Critical Care, Massachusetts General Hospital, Division of Health Sciences and Technology, Harvard Medical School-MIT, Boston, MA 02114, USA*

^b*Center for Learning and Memory, RIKEN-MIT Neuroscience Research Center, Department of Brain and Cognitive Sciences, Massachusetts Institute of Technology, Cambridge, MA 02139, USA*

^c*School of Electrical and Telecommunications, University of New Wales, Sydney, Australia*

Abstract

The spiking activity of rat CA1 hippocampal place cells during open field foraging can be described with stimulus-response models based on an inhomogeneous Poisson (IP) interspike interval probability model. The spatial structure of this model has been previously represented as a simple Gaussian surface. We analyze four new spatial models, two extensions of the Gaussian surface, a surface based on a logistic transformation, and a surface constructed by using Zernike polynomials. Goodness-of-fit analysis based on the Bayesian information criterion (BIC) shows that the Zernike polynomial surfaces give the most accurate description of the spatial place cell spiking activity under this experimental paradigm. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Hippocampal place cells; Interspike interval distribution; Inhomogeneous Poisson process; Zernike polynomials

1. Introduction

As a rat moves freely throughout its environment, pyramidal cells in the CA1 region of the hippocampus demarcate their own areas of elevated firing, termed a place field

* Corresponding author. Neuroscience Statistics Research Laboratory, Department of Anesthesia and Critical Care, Massachusetts General Hospital, Clinics 3, 55 Fruit Street, Boston, MA 02114-2698, USA. Tel.: +1-617-724-1061; fax: +1-617-726-8410.

E-mail addresses: barbieri@SRLB.mgh.harvard.edu (R. Barbieri), loren@neurostat.mgh.harvard.edu (L.M. Frank), mquirk@ladyday.mwl.ai.mit.edu (M.C. Quirk), vic@nmr.mgh.harvard.edu (V. Solo), wilson@cortical.mit.edu (M.A. Wilson), brown@SRLB.mgh.harvard.edu (E.N. Brown).

[5,6]. In the center of a circular environment the place fields resemble Gaussian surfaces whereas when the animal runs closer to the walls, the fields are more asymmetric. We previously modeled the interspike interval (ISI) distribution of CA1 neurons under this paradigm as inhomogeneous Poisson (IP) processes in which the spatial dependence was represented by using a two-dimensional Gaussian surface [1,4].

The objective of this research is to assess the ability of four new parametric surfaces to describe the spatial structure of place fields under the IP assumption. They are: two extensions of the simple Gaussian surface, a surface based on a logistic transformation, and a surface constructed by using Zernike polynomials. Each of the new models is compared to the simple Gaussian model previously considered [1,4] by goodness-of-fit analysis based on the Bayesian information criterion (BIC).

2. Methods

2.1. Experimental methods

Microdrive arrays housing twelve tetrodes were implanted in the CA1 layer of a Long Evans rat (Charles River Laboratories, MA) foraging for randomly scattered chocolate pellets in a black cylindrical environment 70 cm in diameter with 30 cm high walls [6]. Simultaneous recordings of the animal's position, and spiking activity of the CA1 neurons were made during a 23-min foraging period [1]. Twenty-eight of the place cells recorded in the experiment were used in this analysis.

2.2. Statistical methods

The interspike interval probability density for t_k given t_{k-1} associated with the IP probability model is

$$f(t_k | t_{k-1}, \xi) = s(t_k | \xi) \exp \left\{ - \int_{t_{k-1}}^{t_k} s(u | \xi) du \right\}, \quad (1)$$

where $s(t | \xi)$ is the spatial intensity function, or rate function, and ξ is the vector of model parameters.

The intensity function $s(t | \xi)$ which relates the input (position) to the probability structure was previously modeled as a simple two-dimensional Gaussian surface with five parameters: μ_1 and μ_2 the coordinates of the field's center, σ_1 and σ_2 the scale parameters, and α the log of the maximum firing rate. Therefore we have $\xi = [\alpha, \mu_1, \mu_2, \sigma_1, \sigma_2]$ [1,4]. We here consider four new formulations of the intensity function.

Directional Gaussian surface: We introduce a cross-term (one additional parameter $-1 < \rho < 1$) to the simple Gaussian surface model to account for directionality. This yields

$$s(t | \xi) = \exp \left\{ \alpha - \frac{1}{2} \begin{bmatrix} x_1(t) - \mu_1 \\ x_2(t) - \mu_2 \end{bmatrix}' \begin{bmatrix} \sigma_1^2 & \rho \sigma_1 \sigma_2 \\ \rho \sigma_1 \sigma_2 & \sigma_2^2 \end{bmatrix}^{-1} \begin{bmatrix} x_1(t) - \mu_1 \\ x_2(t) - \mu_2 \end{bmatrix} \right\}, \quad (2)$$

where $\xi = [\alpha, \mu_1, \mu_2, \sigma_1, \sigma_2, \rho]$. The simple Gaussian is a special case of the directional Gaussian with $\rho = 0$.

Two-Gaussian surface: Next we consider a function composed of the sum of two-Gaussian surfaces. The intensity function is

$$s(t | \xi) = \exp \left\{ \alpha - \frac{1}{2} \left[\left(\frac{x_1(t) - \mu_1}{\sigma_1} \right)^2 + \left(\frac{x_2(t) - \mu_2}{\sigma_2} \right)^2 \right] \right\} + \exp \left\{ \alpha^* - \frac{1}{2} \left[\left(\frac{x_1(t) - \mu_1^*}{\sigma_1^*} \right)^2 + \left(\frac{x_2(t) - \mu_2^*}{\sigma_2^*} \right)^2 \right] \right\}, \quad (3)$$

where $\xi = [\alpha, \mu_1, \mu_2, \sigma_1, \sigma_2, \alpha^*, \mu_1^*, \mu_2^*, \sigma_1^*, \sigma_2^*]$.

Logistic transformation surface: Our third model is derived by applying a logistic transformation to the coordinates of the simple Gaussian surface. This transformation constrains the estimated field to lie within the 70×70 cm square containing the circular environment. The formulation is

$$s(t | \xi) = \exp \left\{ \alpha - \frac{1}{2} \left[\left(\frac{\log \left(\frac{x_1(t) - a}{b - x_1(t)} \right) - \log \left(\frac{\mu_1 - a}{b - \mu_1} \right)}{\sigma_1} \right)^2 + \left(\frac{\log \left(\frac{x_2(t) - c}{d - x_2(t)} \right) - \log \left(\frac{\mu_2 - c}{d - \mu_2} \right)}{\sigma_2} \right)^2 \right] \right\}, \quad (4)$$

where $a = c = 0$ are the lower boundary coordinates and $b = d = 70$ the upper boundary coordinates in cm, and $\xi = [\alpha, \mu_1, \mu_2, \sigma_1, \sigma_2]$.

Zernike polynomials surface: The Zernike circle polynomials are a complete orthogonal set over the interior of the unit circle. A special case of the Jacobi polynomials, the Zernike polynomials are widely used in optics to describe wavefront aberrations [2]. The general formulation of the Zernike polynomials is

$$Z_n^m(r, \phi) = \begin{cases} R_n^m(r) \sin(m\phi) & m > 0 \\ R_n^m(r) \cos(m\phi) & m < 0 \end{cases} \quad 0 \leq r \leq 1, \quad 0 \leq \phi \leq 2\pi,$$

$$R_n^m(r) = \begin{cases} \sum_{l=0}^{(n-|m|)/2} \frac{(-1)^l (n-l)!}{l! \left(\frac{n+m}{2} - l\right)! \left(\frac{n-m}{2} - l\right)!} r^{n-2l} & (n-m) \text{ even} \\ 0 & (n-m) \text{ odd} \end{cases} \quad n \geq |m| \geq 0, \quad (5)$$

where ϕ is the azimuthal angle, r the radial distance, n the order of the polynomial, and $m = 0, \pm 1, \dots, \pm n$, $r = \sqrt{\tilde{x}_1^2 - \tilde{x}_2^2}$, $\phi = \tan^{-1}(\tilde{x}_2/\tilde{x}_1)$, $\tilde{x}_1 = (x_1 - c_1)/R$, $\tilde{x}_2 = (x_2 - c_2)/R$, $c_1 = c_2 = R = 35$ cm.

The number of radial polynomials for any order n is $2n + 1$. We consider the first 10 non-zero polynomials, up through order $n = 3$. The intensity function is the exponential of the sum of all polynomials weighted by the parameters $k_{n,m}$

$$s(t | \xi) = \exp \left\{ \sum_{n=0}^3 \sum_{m=-n}^n k_{n,m} Z_n^m(r(t), \phi(t)) \right\} \quad (6)$$

and the ten-dimensional parameter vector of the non-zero polynomial coefficients is $\xi = [k_{0,0}, k_{1,1}, k_{1,-1}, k_{2,2}, k_{2,0}, k_{2,-2}, k_{3,3}, k_{3,1}, k_{3,-1}, k_{3,-3}]$.

For each model ξ is estimated from the experimental data by maximum likelihood, and the Bayesian information criterion (BIC) is used to assess goodness-of-fit. BIC gives a single number summaries of the efficiency with which a given model describes a set of data by measuring the trade-off between the decrease in the $-2 \log$ likelihood and the increase in the number of parameters required to achieve that decrease [3].

3. Results

We first compared each of the four new models to the simple Gaussian model previously considered [3,4]. Table 1, column 1 shows for each model the number of cells out of 28 whose goodness-of-fit was better than the simple Gaussian, as measured by BIC. The directional Gaussian model gave a better fit than the simple Gaussian for 5 cells, the sum of two Gaussians for 13 cells, and the logistic transformation for only two cells. For 24 of the 28 cells, the Zernike polynomial model fit better than the simple Gaussian.

We also used BIC to determine which model fit best for each cell. Table 1, column 2 shows the number of cells for which each model gave the best overall fit. Twenty-one of the cells were best fit by the Zernike model, five by the sum of two Gaussians, one by the logistic transformation, and one by the simple Gaussian model. Overall 27 of the 28 cells were fit better by one of the new intensity models. The cell for which the simple Gaussian model was the best fit had the lowest number of spikes, 111.

Table 1
Comparison by BIC of the new four spatial models to the simple Gaussian model previously considered in [2,3]

Spatial structure	Better than Gaussian	Best fit
Gaussian	—	1
Directional	9	0
Two-Gaussians	13	5
Logarithmic	2	1
Zernike	24	21

Numbers in column 1 show, for each model, the number of cells, out of 28, which had a better goodness-of-fit than the simple Gaussian. Column 2 shows the number of cells best fit by each model.

4. Discussion

The new spatial intensity functions showed that the asymmetric structure of the CA1 place fields could be well described by parametric models. In fact, together the two-dimensional Gaussian surface with directionality and the sum of two-Gaussian intensities described better the spatial structure of 18 cells. Second, a limitation of all these Gaussian surfaces is that they are not constrained to lie within the circular environment. Consideration of the logistic transformation function allowed for construction of bounded place fields, but this constraint did not prove to be effective in significantly improving the fit to the data for any appreciable number of cells. Third, use of the Zernike polynomials gave the most improvement in goodness-of-fit for the majority of the cells.

The spatial structures of the Zernike polynomials all lie within the environment and are highly flexible using only a few simple basis functions. An example for one cell is shown in Fig. 1, where it is clear that the place field obtained by a linear combination of only ten polynomials describes the spatial structure of the spiking activity better than the two-dimensional Gaussian surface. The choice of number of polynomials, and therefore the number of parameters, was arbitrary. Further research is required to determine the optimal number of basis functions. This may yield an even better characterization of the place fields. We stress that our goodness-of-fit measure, BIC, penalizes the number of parameters. For the majority of the cells, the improvement in the fit by the Zernike surfaces compared to the simple Gaussian structure outweighed the fact that the former are ten parameters instead of five.

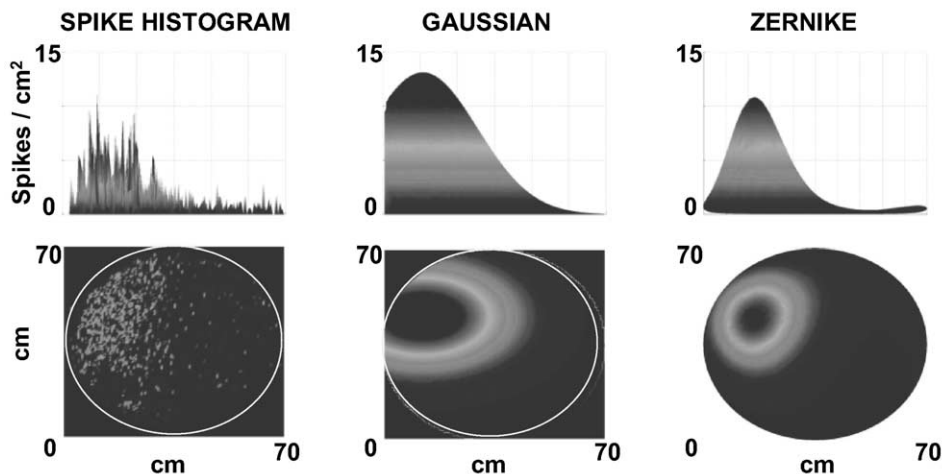


Fig. 1. We model the intensity function as the exponential of the sum of the 10 Zernike polynomials up to the 3rd order. The resulting spatial function is able to shape both symmetric place fields, as well as very asymmetric fields closer to the walls.

Finally, these results, along with our previous work, suggest that both the temporal and the spatial structures are important in the statistical description of how place cell spiking activity depends on position [1]. The history-dependent models (inhomogeneous gamma and inhomogeneous inverse Gaussian) with simple Gaussian spatial structure achieved a greater improvement in fit than the spatial improvement described by the IP model with the spatial structure represented by the Zernike polynomials. Application of both the history-dependent temporal structure and the spatial Zernike surfaces considered together may yield to a further improvement of the fit.

5. Conclusions

Our statistical paradigm allows construction of non-Gaussian surfaces to model the spatial intensity function of the place fields. The new spatial models improved the fit of 27 out of 28 cells considered when compared to the simple Gaussian model. The spatial model based on the Zernike polynomials gave the best overall improvement. A correct spatial model for the single place cell, considered together with history-dependent temporal structures, may lead to more accurate representation of spatial information by single neurons and neural ensembles.

Acknowledgements

This research was supported in part by NIH grants MH59733 and MH61637, and NSF grant IBN 0081548.

References

- [1] R. Barbieri, L.M. Frank, M.C. Quirk, M.A. Wilson, E.N. Brown, Construction and analysis of non-Poisson stimulus-response models of neural spiking activity, *J. Neurosci. Meth.* 105/1 (2001) 25–37.
- [2] M. Born, E. Wolf, *Principles of Optics*, 6th Edition, Pergamon, New York, 1989, pp. 459–490 (Chapter 9).
- [3] G.E.P. Box, G.M. Jenkins, G.C. Reinsel, *Time Series: Forecasting and Control*, 3rd Edition, Prentice-Hall, Englewood Cliffs, NJ, 1994.
- [4] E.N. Brown, L.M. Frank, D. Tang, M.C. Quirk, M.A. Wilson, A statistical paradigm for neural spike train decoding applied to position prediction from ensemble firing patterns of rat hippocampal place cells, *J. Neurosci.* 18 (1998) 7411–7425.
- [5] J. O’Keefe, J. Dostrovsky, The hippocampus as a spatial map: preliminary evidence from unit activity in the freely-moving rat, *Brain Res.* 34 (1971) 171–175.
- [6] M.A. Wilson, B.L. McNaughton, Dynamics of the hippocampal ensemble code for space, *Science* 261 (1993) 1055–1058.